MATH REVIEW

Ω = No of possible configurations

$\Omega = \Omega_1 \cdot \Omega_2$

$S = k \ln \Omega$

Likelihood $\propto \Omega$

$dS = \frac{dE}{T} + \frac{p\,dV}{T} - \frac{\mu\,dN}{T}$ for a reversible change

$$\left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{1}{T} \quad ; \quad \left(\frac{\partial S}{\partial V}\right)_{E,N} = \frac{P}{T} \quad ; \quad \left(\frac{\partial S}{\partial N}\right)_{E,N} = -\frac{\mu}{T}$$

$f = f(x,y)$

$$\delta f = \left.\frac{\partial f}{\partial x}\right|_y \delta x + \left.\frac{\partial f}{\partial y}\right|_x \delta y$$

$$\frac{\partial f}{\partial g} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial g} = \frac{\partial f/\partial x}{\partial x/\partial g}$$

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + \cancel{v}\frac{du}{dx}$$

No. of ways of picking n objects from total of N objects $= {}^N C_n = \frac{N!}{n!(N-n)!}$ gives Binomial distribution.

Stirling's approximation : $N! = (N/e)^N$ for $N \to \infty$

or $\ln N! = N \ln N - N$

No of ways of dividing N objects into r groups of $n_1, n_2, \ldots n_r$ objects

$$\Omega = \frac{N!}{n_1!\, n_2! \ldots n_r!}$$ $\Rightarrow$ gives multinomial distribution

$S = -Nk \sum_{i=1}^{r} p_i \ln p_i$ where $p_i = \frac{n_i}{N}$ (n_i → no of times i^{th} outcome observed; N → trials)

Most likely state of system : Maximize S w.r.t $\{p_1, p_2, \ldots\}$ such that $\sum_i p_i = 1$

or Maximize Ω w.r.t. $\{p_1, p_2, \ldots\}$

Under constraint specified by $g(x_1, x_2, \ldots x_r) = \text{constant}$
a function $f(x_1, x_2, \ldots x_r)$ is maximized by taking

$$\frac{\partial f}{\partial x_i} - \lambda \frac{\partial g}{\partial x_i} = 0 \quad \text{for all } i = 1, 2, \ldots r-1$$

λ is a constant called the lagrange multiplier that can be evaluated from one of the equations/constraints

For multiple constraints

$$\frac{\partial f}{\partial x_i} - \alpha \frac{\partial g}{\partial x_i} - \beta \frac{\partial h}{\partial x_i} \ldots\ldots = 0 \quad \text{for all } i = 1, 2, \ldots$$

Second law $\quad \Delta S_{composite} > 0$

or $\Delta S_{system} + \Delta S_{bath} > 0$

For reversible processes $\quad \Delta S_{composite} = 0$

$\Delta S_{system} = -\Delta S_{bath}$

Gaussian function $\quad \frac{1}{(2\pi\sigma^2)^{1/2}} \exp^{-\frac{(x - x_c)^2}{2\sigma^2}}$

For a probability distribution $p(x)$

Normalization requires $\int_{-\infty}^{\infty} p(x)\, dx = 1$

Average $\hat{x} = \int_{-\infty}^{\infty} x\, p(x)\, dx$

Std. deviation $= \sqrt{\int_{-\infty}^{\infty} (x - \hat{x})^2\, p(x)\, dx}$

Maximum value is where $\frac{dp(x)}{dx} = 0$ and $\frac{d^2 p(x)}{dx^2} < 0$

At equilibrium $\quad \frac{1}{T_1} = \frac{1}{T_2} \;;\; \frac{P_1}{T_1} = \frac{P_2}{T_2} \;,\; \frac{\mu_1}{T_1} = \frac{\mu_2}{T_2}$

Driving forces $\quad \frac{1}{T_1} - \frac{1}{T_2}$ for dE transfer (heat flow)

$\frac{P_1}{T_1} - \frac{P_2}{T_2}$ for dV transfer (expansion)

$\mu_1/T_1 - \mu_2/T_2$ for $-dN$ transfer (mass exchange)

Some integrals

$$\int e^x dx = e^x$$

$$\int x e^x dx = e^x (x-1)$$

Some series

$$1 + r + r^2 + \ldots = \frac{1}{1-r} \quad \text{when} \quad |r| < 1$$

Taylor's expansion

$$f(x) = f(x = x_0) + \left.\frac{df}{dx}\right|_{x=x_0} (x - x_0) + \frac{1}{2!} \left.\frac{d^2 f}{dx^2}\right|_{x=x_0} (x-x_0)^2 + \frac{1}{3!} \left.\frac{d^3 f}{dx^3}\right|_{x=x_0} (x-x_0)^3 + \ldots$$